INITIAL STAGES IN THE DEVELOPMENT OF THERMALS

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It is well known that a free spherical volume of fluid that is lighter than the surrounding medium is transformed into a vortex ring, i.e., the core of a buoyant vortex ring. References [1-6] are devoted to the study of such vortices at different stages of motion. The least studied is the initial stage of the development, viz., the very transformation process. In the present paper the transformation process is discussed, experimental results are given, and equations are developed to determine the fundamental parameters that arise as a result of such a process of the buoyant vortex-thermal ring.

1. Let R_0 be the effective radius of the initial volume of the thermal $Q_0 = 4\pi R_0^3/3$; ρ_1 is the density of the fluid inside Q_0; ρ_0 is the density of the external medium, p is the pressure, Γ is the circulation, H is the height to which it rises; R, r are the radii of the axial circle and the toroidal section of the vortex ring core, $\xi = (\rho_0 - \rho_1)/\rho_0$ is the relative density, and $\alpha = dR/dH$ is the aperture angle. Following [1], we introduce nondimensional quantity $H^\circ = H/R_0$, $\Gamma^\circ = \Gamma/R_0\sqrt{R_0 g}$.

The average value of the parameter $\alpha = 0.25$ [2]. Experiments, based on which this value has been obtained, were conducted in water for small ξ, where water with higher density ($\xi < 0$) than the external medium overflowed into a reservoir from the vessel. Under these conditions the motion of the descending mass resembles the motion of Hill's vortex [2].

Experiments conducted with thermals [5] formed during the collapse of a film of soap bubble filled with lighter-than-air gas gives a different picture of the motion. In these experiments (results taken from this work are given in Fig. 1 and 2) the mean value of α = 0.09 for all $\xi > 0$, whereas, as shown by experiments, if the axial symmetry of the motion is not appreciably altered, not only a change in ξ but also artificially introduced disturbances (additional initial vorticity, collapse of the film when punctured at the side or bottom) have very little effect on the value of α.

All these facts indicate that the final result of the transformation process of the volume Q_0 to the toroid, i.e., the vortex ring core, is determined by certain integral relations and does not depend on arbitrary factors.

2. Consider the problem for an ideal fluid. Let at time t = 0 the spherical volume Q_0 have free boundaries. Under the action of buoyancy $F = Q_0(\rho_0 - \rho_1)g$ the thermal starts rising with an acceleration [8].

$$\frac{d^2H}{dt^2} = \frac{Q_0(\rho_0-\rho_1)g}{(Q_0\rho_1 + 0.5Q_0\rho_0)} = \frac{2\xi g}{3-2\xi}.$$

At the initial moment of time the pressure distribution outside Q_0 is determined by the Lagrangian equation

$$p|_{t=0} = p_\infty - \rho_0\varphi_t + \rho_0 hg,$$

where h is the difference in height between the top of Q_0 and a certain reference point; φ is the velocity potential; p_∞ is the hydrostatic pressure at h = 0. Similarly, inside the thermal $p_1|_{t=0} = p_\infty - \rho_1\varphi_{1t} + \rho_1 hg$.

For the flow outside Q_0 it is now possible to use the velocity potential for the flow around a sphere [8] which has the form $\varphi = 1.5vR_0\cos\theta$, at the points on the thermal surface, here θ is the polar angle ($h = 2R_0\sin^2(\theta/2)$); v = v(t) is the velocity of the external flow around the thermal. Inside Q_0 at small $t(t\to 0)$ φ_{1t} — is a function of time only.

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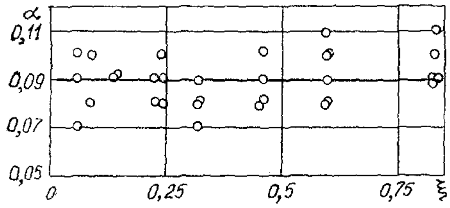

Fig. 1

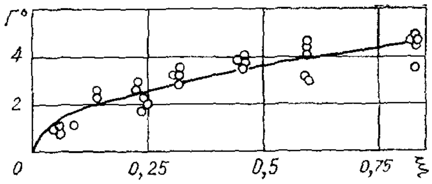

Fig. 2

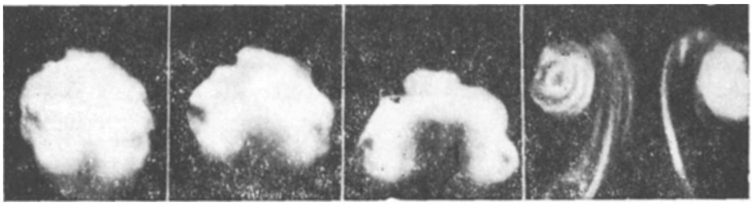

Fig. 3

As the thermal rises, vorticity is generated in the region adjacent to its boundary, and consequently, there is circulation. Its increase is determined by Ber'knes' theorem [6, 7] $d\Gamma/dt = -\oint dp/\rho$.

We integrate it along the following contour: from the upper point of Q_0 along the meridian to the lower point and back along the vertical in the axis of symmetry. As a result we have

$$d\Gamma/dt|_{t=0} = -2R_0g - \varphi_t(\pi) + \varphi_t(0) + 2R_0g.$$

Substituting the quantity φ_t in this equation and considering that at $t = 0$, $dv(0)/dt = d^2H/dt^2$, we get

$$d\Gamma/dt|_{t=0} = 3R_0d^2H/dt^2 = 6\xi gR_0/(3 - 2\xi),$$

hence for small t we have

$$\Gamma = \frac{6\xi gR_0}{3 - 2\xi}t. \qquad (2.1)$$

It follows from the analysis that this equation determines the increase in Γ as long as there is no appreciable deformation of the boundaries of Q_0 (apparently, until the "puncturing" of the thermal by the external medium).

Let us estimate the time to attain the experimentally observed circulation (see Fig. 2). For $\xi = 0.87$, $R_0 = 5$ cm, $\Gamma^\circ \sim 5$, according to (2.1) $t = 0.08$ sec. As shown by photographs of the thermal motion its deformation is small during this period. Vorticity diffuses into the depth of Q_0 and during the motion it spreads everywhere in the volume. However, it cannot be the cause of the transformation of the spherical volume until that time.

3. The transformation process should be accompanied by the appearance of motion of the external medium at the lower part of Q_0 and directed towards its center ("reverse flow"), which could be caused only by the associated pressure gradient. The motion inside Q_0 does not significantly affect its magnitude because of the relatively small value of the relative density ($\rho_1 \quad \rho_0$). The "reverse" gradient is created by the difference in hydrostatic pressures inside and outside the thermal at the same height. This difference $\Delta p = (\rho_0 - \rho_1)gh\xi$ at $t = 0$ is compensated by inertial forces, and further in the upper half by the

dynamic pressure $\rho_0 V^2/2$ of the flow past Q_0. In the lower half, where such a compensation is not possible since the magnitude of $\rho_0 V^2/2$ after passing through the maximum at $h \simeq R_0$, unlike Δp, starts decreasing and the compensation is achieved by axial "reverse" flow of the external medium. Let its velocity at the thermal axis be u. At the central part of the thermal $u = u_c$ is directed along the vertical, and is determined by the height $\Delta h = R_0$, hence

$$\rho_0 u_c^2/2 \approx R_0 g \rho_0 \xi, \tag{3.1}$$

from which $u_c = \sqrt{2gR_0\xi}$.

Let us compare the value of u_c with the velocity u_h at the center of Hill's vortex. This velocity is associated with the vortex motion w_h as a whole by the relation [8] $w_h = 2u_h/3$.

The circulation of Hill's vortex is determined by the equation [8] $\Gamma = 5w_h R_0$.

Assuming $u_h = u_c = \sqrt{2gR_0\xi}$, we get an estimate for

$$\Gamma = 5(2/3)R_0\sqrt{2gR_0\xi}$$

or

$$\Gamma^0 = 4.72\sqrt{\xi} \sim 5\sqrt{\xi} \tag{3.2}$$

(curve computed on the basis of Eq. (3.2) is shown plotted in Fig. 2).

We note that the use of the particular solution of the equation of motion for Hill's vortex in order to determine Γ is not essential. The energy relation (3.1) between the reverse velocity u_c and the potential energy that ensures this velocity is essential. The order of magnitude of circulation can also be determined from the equation

$$\Gamma = 2\pi u_c R_0/2 \simeq \pi R_0 \sqrt{R_0 \xi g},\ \Gamma^0 \simeq \pi\sqrt{2\xi} \sim 5\sqrt{\xi}.$$

Here the toroid inscribed in the sphere of radius R_0 has been taken with parameters $R = R_0$, $r = R_0/2$.

In light of the above considerations it is interesting to refer to the photographic frames of the thermal motion during the initial stages of its development. Four frames from such a film are shown in Fig. 3. They are obtained by freezing the vertical section of the thermal by the camera. It is visualized by an optical knife formed by a beam of argon laser through a convex cylindrical mirror.

The initial volume Q_0 is filled with a carefully prepared mixture of helium and nitrogen with a small quantity of oxygen. The relative density drop for this thermal $\xi = 0.7$, $R_0 \sim 5$ cm.

As the mixture was filled it was lightly colored by cigarette smoke. A certain (small) amount of the mixture was introduced without coloring. In Fig. 3 the volumes that last enter the plane of the optical knife are seen in the form of dark spots on the illuminated section of the thermal. The time interval between the exposure of the first three frames is 0.04 sec, between the third and the fourth frames it is 0.3 sec. These photographs confirm the suggested model for the motion which can be qualitatively described as follows.

The uncompensated pressure Δp at the initial instant of time, as mentioned earlier, is balanced by inertial forces. Then the thermal surface at the lower part begins to collapse. Conditions and the nature of the flow in this region of the thermal resembles the motion during spherical accumulation [9]. The lower part of the surface is pushed inside Q_0. The particles of the external medium following it form an axial jet. All these phases of the thermal development are seen in the first three frames of Fig. 3. A sharp intrusion of the dense jet inside Q_0 in certain cases can even lead to the expulsion of a small part of the gas occupying the thermal through its roof in the form of short jets. Simultaneously a rapid increase in vorticity takes place at the thermal surface. In view of this, we observe from the first three frames of Fig. 3 the rotation of the dark oblong spot on the left of the section, i.e., the rotation of the colorless volume of the mixture near the thermal surface.

Thus, vortex generation occurs due to noncollinearity at the beginning of the motion of isochore and isobars (Ber'knes' theorem) at the thermal surface, the transformation into toroid takes place due to the presence of grad ΔP at the initial stages of motion.

Experiments show that the buoyant vortex ring formed in such a manner consists of a toroidal core where the fluid from the volume Q_0 goes (photograph of its section is shown in the fourth frame in Fig. 3) and the external potential flow.

The circulation computed from (3.2) is close to the maximum, since $\Delta h = R_0$ is the maximum dynamic head. If, at the beginning of the motion vorticity is present for any reasons at the boundary or inside the thermal and if its strength exceeds the above value of Γ, then in this case, apparently, in order to balance the flow inside and outside the toroidal core the "extra" vorticity is completely or partially dissipated into the external medium.

4. Relations coupling circulation Γ to the angle α can also be approximately determined for the developing thermal. The developing, buoyant vortex ring has a potential energy Π and kinetic energy E_k. The former is completely determined by the weight deficit $F = Qg\xi(\rho_0 - \rho_1)$, $\Pi = FH$, where Q is the volume of the vortex ring core filled with a fluid lighter than the surrounding medium. At the initial stage of motion $Q = Q_0$.

Kinetic energy E_k is determined by the circulation and geometric parameters of the core and is described by the approximate formula

$$E_{\kappa} = 0.5\Gamma^2 R\rho_0\,[\ln(8R/r) - 2], \tag{4.1}$$

obtained in [10] with the assumption that Bio–Savart law is applicable and that the ratio r/R is small for vortex rings in an ideal fluid. We shall assume that Γ and $Q = 2\pi^2 Rr^2$ remain constant during the process and as a result of buoyancy (reduction in Π) there is an increase in R and a corresponding decrease in r. In this case, differentiating (4.1) with respect to t with Q = const. and equating the result to $d\Pi/dt$, we get

$$\Gamma^0 = 4\sqrt{\frac{\pi\xi}{3\alpha\,[2\ln(8R/r) - 1]}}$$

$$\alpha = \frac{16\pi\xi}{3\Gamma^2\,[2\ln(8R/r) - 1]}. \tag{4.2}$$

Substituting (3.2) in (4.2) we get

$$\alpha \simeq \frac{16\pi}{75\,[2\ln(8R/r) - 1]}. \tag{4.3}$$

The ratio r/R obtained from the analysis of photo frames of the thermal motion, lies in the range 0.1-0.2. For these values of r/R it follows from (4.3) that $\alpha \approx 0.09$-0.1 (good agreement with experiment (see Fig. 1)).

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